Opener

If f is a differentiable function, then $f'(a)$ is given by which of the following?

I. $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

II. $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

III. $\lim_{x\to a} \frac{f(x+h)-f(x)}{h}$

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

Sep 18-3:22 PM

3-2 Differentiability

Learning Objectives:

I understand different way that a function might be non-differentiable.

I understand how to find/graph derivatives on a graphing calculator at a given x.

I understand that differentiability implies local linearity and continuity.

I can understand the Intermediate Value Theorem for derivatives.

Sep 18-3:22 PM

One Sided Derivatives

A function y = f(x) is differentiable (the derivative exists) at a point x = c if and only if

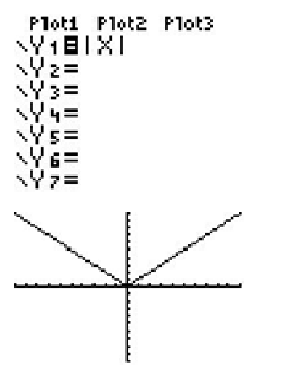
$$f'(x) = \lim_{x \to c+} \frac{f(x) - f(c)}{x - c} = \lim_{x \to c-} \frac{f(x) - f(c)}{x - c}$$

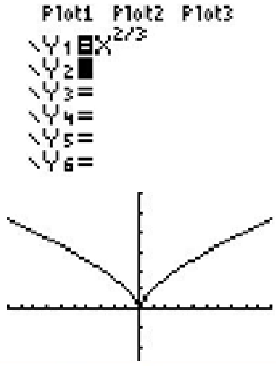
In other words, the slope must be approaching the same thing on the left side as it is on the right side. If there is an abrupt change in the slope at some point x = c, that means that the function is non-differentiable at that point.

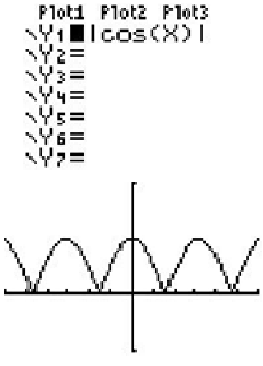
How could a function be Non-Differentiable? What would it look like?

Sep 18-3:22 PM

Corners and Cusps



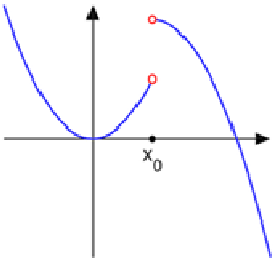


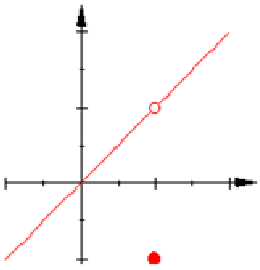


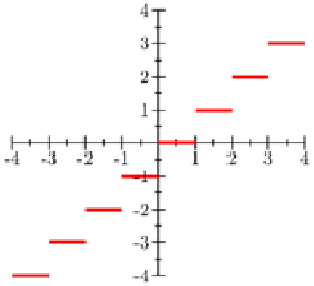
Remember, the derivative is really the slope. The slope is not approaching the same thing on both sides.

Sep 18-3:22 PM

Discontinuities



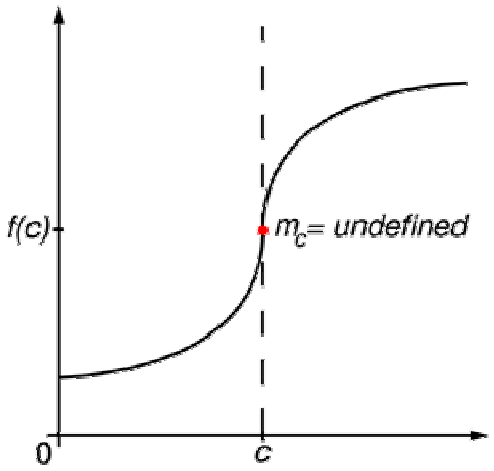




Remember, a derivative is really the slope of a tangent line. If there is not tangent line, there is no slope, there is no derivative.

Sep 18-3:22 PM

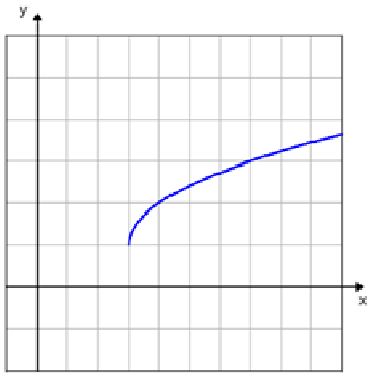
Vertical Tangent Lines

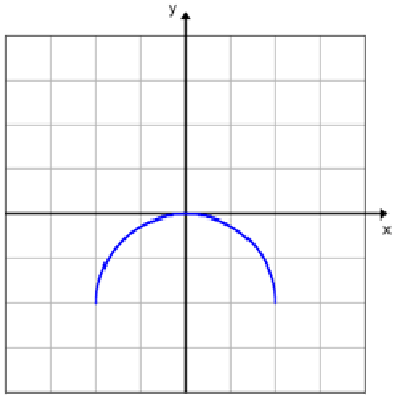


Remember, a derivative is really a slope. A vertical tangent line has an undefined slope hence the derivative is undefined too. This case is different than the others in that there actually is a tangent line at the point in question – its just that the slope of that tangent line isn't defined.

Sep 18-3:22 PM

Endpoints

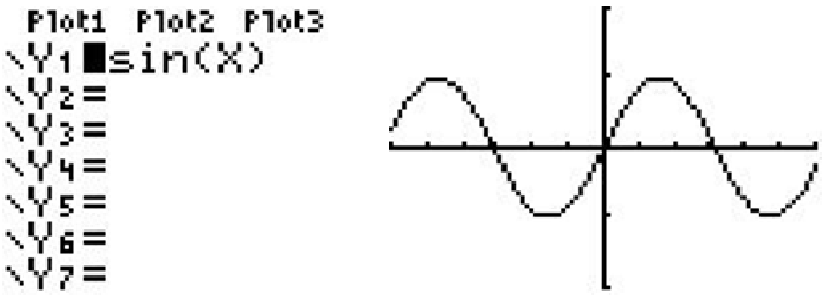




Remember, a derivative is really the slope of a tangent line. If there is not tangent line, there is no slope, there is no derivative. The slope exists on one side but not the other.

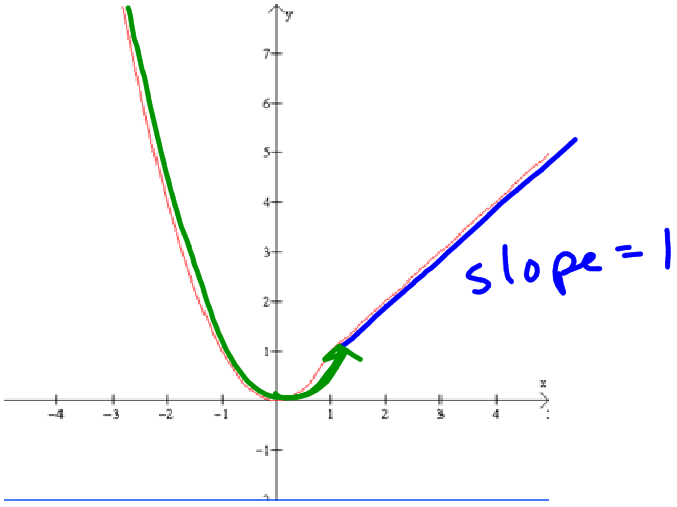
Sep 18-3:22 PM

Differentiability Implies Local



Sep 18-3:22 PM

$$f(x) = \begin{cases} x^2 & if \quad x \le 1 \\ x & if \quad x > 1 \end{cases}$$



Ex1

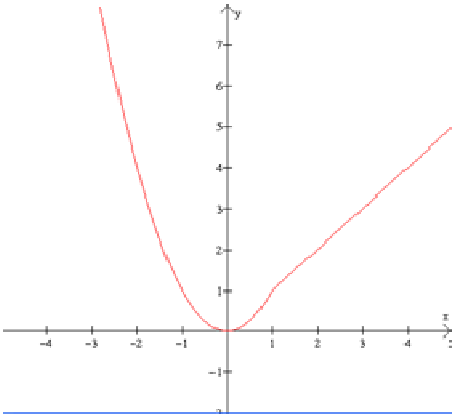
a.) Do you think that this function is differentiable at x = 1? Why or why not?

b.) Find the right hand and left hand derivatives at x=1.

Sep 18-3:22 PM

Since, the slopes are approaching different values on the left and right side of x = 1, the function is not differentiable at x = 1.

$$f(x) = \begin{cases} x^2 & if \quad x \le 1 \\ x & if \quad x > 1 \end{cases}$$

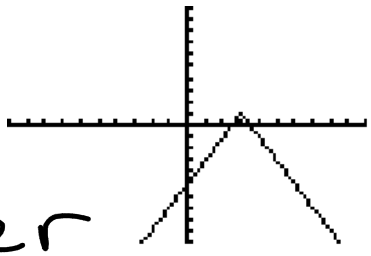


c.) Make it so that this function is differentiable at x = 1. You may only change 1 thing in the function.

Sep 18-3:22 PM

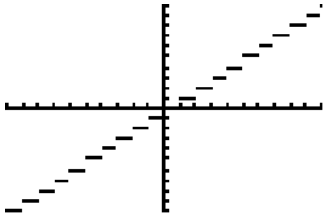
Ex2. Find all points in the domain of f(x) for which f(x) is NOT differentiable. Identify why the function is not differentiable at each of these points.

1.) $y = -2|x-3|+1$



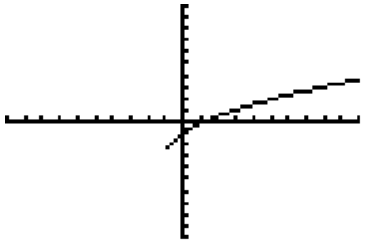
non-diff @ (3,1) corner

2.) $f(x) = [\![x]\!]$



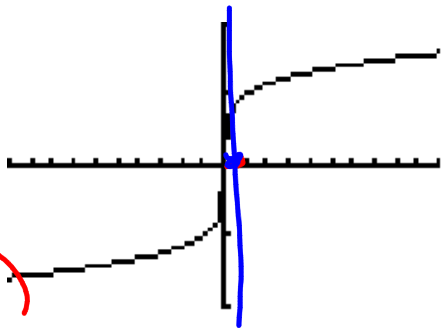
non diff @ all int discontinuit

3.) $g(x) = 2\sqrt{x+1} - 3$



non-diff @ (-1,-3) endpt.

4. $h(x) = \sqrt[5]{x}$



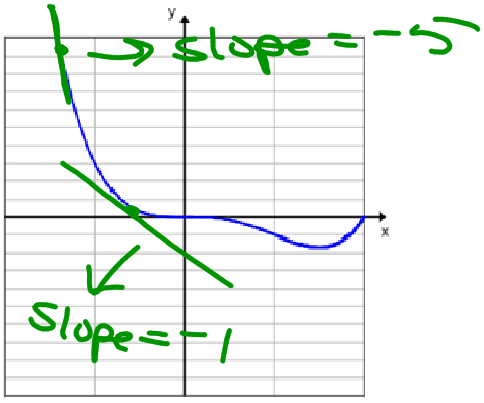
non dif @ (0,0)

vertical tangent line

Sep 18-3:22 PM

Intermediate Value Theorem for Derivatives

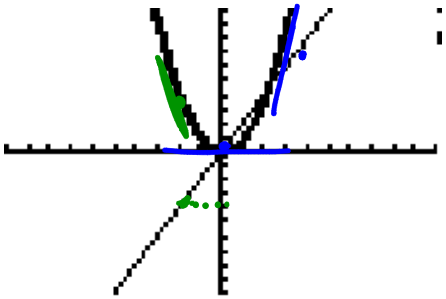
If ***a*** and ***b*** are any two points in an interval on which f is differentiable, then f' takes on every value between f'(a) and f'(b) somewhere on the interval .



Sep 18-3:22 PM

Ex3. Graph the derivative of each function the graphing calculator

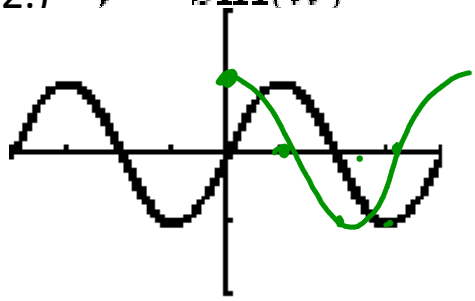
1. $y = x^2$

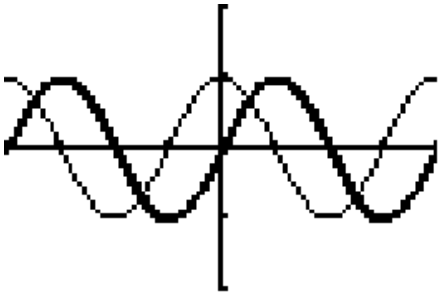


$y' = 2x$

$\frac{d}{dx}(\sin x) = \cos x$

2.) $y = \sin(x)$





Sep 18-3:22 PM

Homework

pg 114 # 1-16, 27-32, 34

Sep 18-3:22 PM